ABSTRACTS OF TALKS

On Green's functions for complex Timoshenko boundary value problems

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We present a uniform and systematic way of construction right inverse operator for fourth order equation with arbitrary complex coefficients of the shape

$$Ly = y^{(4)} - 2py'' + qy = f$$

parameterized by two arbitrary complex constants $\alpha, \beta,\ p = \frac{1}{2}(\alpha^2 + \beta^2), q = \alpha^2\beta^2$. The semigroup generated by the corresponding system can be represented by Kouriensky type functions $Y_6, \ldots, Y_0$ where $Y_4(t-s) = \sum_{n=1}^{\infty}(\alpha^2 - \beta^2)^{-1}(\alpha^2 + \beta^{2n+1})(t-s)^{2n+1}/(2n+1)!$ is Cauchy function (symbolically $(\alpha^2 - \beta^2)^{-1}(\alpha^{-1}\sinh\alpha(t-s)) - \beta^{-1}\sinh\beta(t-s))$.

The representation $(\exp\{tA\})$ is given by the Wronski's matrix of the fundamental system of solutions to the homogeneous equations $(f = 0)$: $\{-qY_5(t), -qY_6(t), Y_3(t), Y_4(t)\}$;

$$\mathcal{G}_t(t,s) + \mathcal{G}_s(t,s) = \mathcal{Y}(t)\mathcal{M}\mathcal{Y}(s)$$

where $\mathcal{M}$ is given by the boundary data eg. for Neumann problem (which describes the beam free on both extremes) the entries of $\mathcal{M}$ are quotients of the wronskian type determinants of $\dot{\mathcal{Y}}(1)$.

Dirichlet problem for nonlocal operators I, II, III

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We solve the extension problem for nonlocal operators, such as the fractional Laplacian in Sobolev spaces, under minimal regularity of the external values. The extension with the smallest Sobolev norm is given by the Poisson integral and coincides with the variational solution of the corresponding Dirichlet problem. The Sobolev norm of the Poisson extension is expressed by a weighted Sobolev norm of the external data.

There will be three lectures on this subject: by Artur Rutkowski (20 min), by Krzysztof Bogdan (40 min), and by Katarzyna Pietruska-Pałuba (60 min).

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On the existence of homoclinic type solutions of inhomogenous Lagrangian systems

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We study the existence of homoclinic type solutions for second order Lagrangian systems of the type $\ddot{q}(t) - q(t) + a(t)\nabla G(q) = f(t)$, where $t \in \mathbb{R}$, $q \in \mathbb{R}^n$, $a : \mathbb{R} \to \mathbb{R}$ is a continuous positive bounded function, $G : \mathbb{R}^n \to \mathbb{R}$ is a C^1-smooth potential satisfying the Ambrosetti-Rabinowitz superquadratic growth condition and $f : \mathbb{R} \to \mathbb{R}^n$ is a continuous bounded square integrable forcing term. A homoclinic type solution is obtained as limit of $2k$-periodic solutions of an approximative sequence of second order differential equations.

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2D Quasi-Geostrophic equation; sub-critical and critical cases

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We are using a version of the 'vanishing viscosity technique' to study the Dirichlet problem for *critical 2D Quasi-Geostrophic equation*:

$$\begin{aligned} &\theta_t + u \cdot \nabla\theta + \kappa(-\Delta)^\alpha\theta = f, \quad x \in \Omega \subset \mathbb{R}^2,\ t > 0, \\ &\theta(0, x) = \theta_0(x), \end{aligned} \tag{0.1}$$

where θ represents the potential temperature, $\kappa > 0$ is a diffusivity coefficient, $\alpha \in [\frac{1}{2}, 1]$ a fractional exponent, and $u = (u_1, u_2)$ is the *velocity field* determined by θ through the relation:

$$u = \mathcal{R}^\perp\theta \quad \text{with } \mathcal{R} = \nabla(-\Delta)^{-\frac{1}{2}}. \tag{0.2}$$

Solving first the *sub-critical approximating equations* (0.1) with $\alpha \in (\frac{1}{2}, 1]$, we let then $\alpha \to \frac{1}{2}$ to obtain a solution corresponding to the critical value of exponent $\alpha = \frac{1}{2}$. We discuss next in some details properties of solutions of the critical problem; in particular their uniqueness, regularity and long time behavior. The lecture is based on the recent publication [1], joint with Chunyou Sun, and uses the technique presented in [2].

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Gradient regularity in Orlicz space for the gradient of solutions to quasilinear elliptic equations in the plane

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For a planar domain Ω, we investigate the Dirichlet problem

$$\begin{cases} -\operatorname{div} A(x, \nabla v) = f & \text{in } \Omega \\ v = 0 & \text{on } \partial\Omega\,, \end{cases}$$

for a quasilinear elliptic equation when f belongs to the Zygmund space $L(\log L)^{\delta}(\log\log\log L)^{\frac{\beta}{2}}(\Omega)$ with $\beta \geq 0$ and $\delta \geq \frac{1}{2}$.

We prove that the gradient of the variational solution $v \in W_0^{1,2}(\Omega)$ belongs to the space $L^2(\log L)^{2\delta-1}(\log\log\log L)^{\beta}(\Omega)$.

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Cohomology height of Conley modules applied to multiple bifurcations for elliptic BVP

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A module structure of Conley index is used to define an invariant called the co-homological height. It is then applied to obtain multiple bifurcations of solutions to some elliptic equations with Dirichlet boundary conditions by use some simple symmetries.

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Hemivariational inequalities modeling electro-elastic unilateral frictional contact problem

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We study a class of abstract hemivariational inequalities in a reflexive Banach space. For this class, using the theory of multivalued pseudomonotone mappings and a fixed point argument, we provide a result on existence and uniqueness of solution. Next, we investigate a static frictional contact problem with unilateral constraints between a piezoelastic body and a conductive foundation. The contact, friction and electrical conductivity condition on the contact surface are described with the Clarke subdifferential multivalued boundary relations. We derive the variational formulation of the contact problem which is a coupled system of two hemivariational inequalities. Finally, for such system we apply our abstract result and prove its unique weak solvability.

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Some bifurcation results for fractional Laplacian problems

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Brezis–Nirenberg theorem and its applications

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In the talk the Brezis–Nirenberg theorem will be presented. It establishes the coincidence of the C^1 minimizer of the functional

$$\Phi(u) = \int_\Omega \frac{1}{2}|\nabla u|^2 - f(x,u)$$

with its H_0^1 minimizer. Some generalizations of this important result will be discussed as well. Finally, we will focus on the role of those theorems in obtaining the global multiple solutions of the corresponding boundary problem with Laplace-type equation.

Radial solutions for nonlinear elliptic equation with nonlinear nonlocal boundary condition

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We consider the problem for the nonlinear elliptic partial differential equation in an annular domain

$$-\Delta u = f(|x|, u), \qquad u|_{\partial\Omega} = \int\limits_\Omega K(|x|,|y|)h(u(x))\,dx, \tag{0.3}$$

where $\Omega = B(0,b) \setminus \overline{B}(0,a) \subset \mathbb{R}^n$, $0 < a < b$.

The problem (0.3) was motivated by applications in a quasi-static theory of thermoelasticity. We look for radial solutions for (0.3). Using appropriate substitutions, the problem (0.3) was reduced to the second-order ordinary differential equation with a couple of two nonlinear nonlocal conditions. Using some fixed point theorem in a cone, we will prove the existence of at least one solution for the "reduced" problem.

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Existence of compactification and applications to Majda DiPerna measures theory

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In the book by Engelking [1] p. 240 there is given the following exercise. Let $\mathbb{X}$ be the Tychonoff space and let $\mathcal{R}$ be the given complete ring of continuous functions defined on $\mathbb{X}$. Prove that there exists such a compactification $\gamma\mathbb{X}$ of set $\mathbb{X}$ that $\mathcal{R}$ can be identified with the set of all continuous functions on $\gamma\mathbb{X}$. This statement has found an astonishingly broad application to the calculus of variations, when considering the ability to control behavior of weak limits of sequences of compositions via Generalised Young Measures, introduced by DiPerna and Majda in [2].

The proof suggested by Engelking is however difficult to follow for non-specialists in topology and is perhaps too general for many applications. This is because it requires introducing topology on the set of ideals of the ring of continuous real valued functions defined on a space $\mathbb{X}$. The space $\mathbb{X}$ itself is only assumed Tychonoff's. Such general approach gives us poor control on separability and metrizability of the compactification $\gamma\mathbb{X}$.

My goal is to present an easy and constructive proof of this fact in the special case, when the abstract Tychonoff's space $\mathbb{X}$ appears to be a subset of $\mathbb{R}^n$. The precise geometric construction allows us to clearly identify the resulting space $\gamma\mathbb{X}$, which under our assumptions appears to be metric and separable.

As an application I obtain certain generalization of DiPerna–Majda theorem (see also for example [3, 4]), dealing with functionals of the form

$$I(u) = \int_\Omega f(x,u)dx$$

where function f is possibly discontinuous with respect to x and u. Such functionals appear for example in the optimal design problem. The statement can be considered as a variant of Convergence Theorem known in set valued analysis.

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Heterogeneous thin films: local and nonlocal effects

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I will discuss a variational model for heterogeneous thin films (membranes) including so-called Cosserat vectors (directors). As it turns out, although the corresponding finite-scale model is a perfectly local integral functional, nonlocal effects can appear in the homogenized thin film limit in some cases. On the other hand, this phenomenon can be ruled out for sufficiently fine heterogeneities.

This is joint work with Carolin Kreisbeck (Utrecht).

Constrained partial differential equations

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When discussing different type of PDEs and systems of PDEs, e.g. of reaction-diffusion diffusion type and their applications, the existence and behavior of their solutions, dynamics and asymptotic properties, and looking onto diverse applications, the natural state constraints come into the sight. For instance if one studies equations describing the concentrations of reacting and substances being subject to diffusion the natural constraint is that, on one hand, they must be nonnegative and, on the other hand, there is an upper bound beyond which substances are saturated or the phase transition takes place. The behavior of natural state constraints has its influence on the reaction (nonlinear term). These issues have interesting impact onto the mathematical model and will be discussed in the talk.

Micropolar meets Newtonian. The Rayleigh–Bénard problem

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We consider the Rayleigh–Bénard problem for the two-dimensional Boussinesq system for the micropolar fluid. Our main goal is to compare the value of the critical Rayleigh number, and estimates of the Nusselt number and the fractal dimension of the global attractor with those values for the same problem for the classical Navier–Stokes system. Our estimates reveal the stabilizing effects of micropolarity in comparison with the homogeneous Navier–Stokes fluid. In particular, the critical Rayleigh number for the micropolar model is larger then that for the Navier–Stokes one, and large micropolar viscosity may even prevent the averaged heat transport in the upward vertical direction. The estimate of the fractal dimension of the global attractor for the considered problem is better then that for the same problem for the Navier–Stokes system. Besides, we obtain also other results which relate the heat convection in the micropolar fluids to the Newtonian ones, among them, relations between the Nusselt number and the energy dissipation rate and upper semicontinuous convergence of global attractors to the Rayleigh–Bénard attractor for the considered model.

Semilinear elliptic problem with nonlinear boundary

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We investigate the existence of solutions for the following class of semilinear elliptic equation with nonlinear boundary

$$\begin{aligned} \Delta x(y) &= F_x\left(y, x(y)\right) - G_x(y, x(y)) \text{ in } \Omega, \\ -\partial_\nu x(y) &= H_x(y, x(y)) - Q_x(y, x(y)) \text{ on } \Gamma, \end{aligned}$$

where Ω is an open bounded domain in $\mathbb{R}^n$, $n \geq 2$, with boundary Γ being sufficiently smooth. Our approach is based on the variational methods.

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The Nitsche phenomenon for weighted Dirichlet energy

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The present paper arose from recent studies of energy-minimal deformations of planar domains. We are concerned with the Dirichlet energy. In general the minimal mappings need not be homeomorphisms. In fact a part of the domain near its boundary may collapse into the boundary of the target domain. In mathematical models of nonlinear elasticity this is interpreted as *interpenetration of matter*. We call such occurrence the *Nitsche phenomenon*, after Nitsche's remarkable conjecture (now a theorem) about existence of harmonic homeomorphisms between annuli. Indeed the round annuli proved to be perfect choices to grasp the nuances of the problem. Several papers are devoted to a study of deformations of annuli. Because of rotational symmetry the Dirichlet energy-minimal deformations turned out to be radial maps. That is why we confine ourselves to radial minimal mappings. The novelty lies in the presence of a weight in the Dirichlet integral. We observe the Nitsche phenomenon in this case as well. However, the arguments require further considerations and new ingredients. One must overcome the inherent difficulties arising from discontinuity of the weight. The Lagrange-Euler equation is unavailable, because the outer variation violates the principle of none interpenetration of matter. Inner variation, on the other hand, leads to an equation that involves the derivative of the weight. But our weight function is only measurable which is the main challenge of the present paper.

Semilinear elliptic equations involving Dirichlet operator and measure data

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Let $(\mathcal{E}, D(\mathcal{E}))$ be a regular Dirichlet form on $L^2(E;m)$, μ be a bounded smooth measure (with respect to the capacity associated with $(\mathcal{E}, D(\mathcal{E}))$ and $f : E\times\mathbb{R}\to\mathbb{R}$ be a measurable function. We consider semilinear equation of the form

$$(*)\qquad -Lu = f(\cdot, u) + \mu \quad \text{in} \quad E,$$

where L is the operator associated with $(\mathcal{E}, D(\mathcal{E}))$. The class of operators associated with Dirichlet forms is quite large. It includes local operators (the model example is the Laplace operator or uniformly elliptic divergence form operator) as well as nonlocal operators (the model example is the fractional Laplace operator).

We first define the probabilistic solution to $(*)$ (roughly speaking, u is a probabilistic solution to $(*)$ if it satisfies some equation, which may be viewed as a nonlinear Feynman–Kac formula associated with $(*)$). Then we show that u is a probabilistic solution if and only if it is a renormalized solution in the sense defined in [3]. This result enables us studying renormalized solutions to $(*)$ by probabilistic methods. By way of illustration what can be proved by these methods, we show some results on existence, uniqueness and regularity of solutions of $(*)$ proved in [2, 4]. These results generalize to the case of Dirichlet operators the corresponding results from [1] proved in the case where L is a uniformly elliptic divergence form operator.

Joint work with Tomasz Klimsiak.

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Some weakly coercive quasilinear problems with forcing

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We consider the forced problem

$$-\Delta_p u - V(x)|u|^{p-2}u = f(x),$$

where $\Delta_p u$ is the p-Laplacian $(1 < p < \infty)$ in a domain $\Omega \subset \mathbb{R}^N$, $V \geq 0$ and

$$Q_V(u) := \int_\Omega |\nabla u|^p \, dx - \int_\Omega V|u|^p \, dx > 0 \quad \text{for all test functions } u \in \mathcal{D}(\Omega) \setminus \{0\}.$$

We show that under some additional conditions on Q_V this problem has a solution for all f in a suitable space of distributions. Then we apply this result to some classes of functions V which in particular include the Hardy potential $V(x) = \left(\frac{N-p}{p}\right)^p |x|^{-p}$ and the potential $V(x) = \lambda_{1,p}(\Omega)$, where $\lambda_{1,p}(\Omega)$ is the Poincaré constant on the strip $\Omega = \omega \times \mathbb{R}^M$, $\omega \subset \mathbb{R}^{N-M}$ bounded.

This is joint work with Michel Willem.

Some observations about supremal functionals and their non-local features

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I will recall the notion of supremal functionals and show the connection with infinity harmonic-type equations, enlightening the non-local aspects. I will also present some recent results I obtained regarding existence of solutions for related minimum problems, emphasizing the connection between supremal functionals and unbounded integral functionals.

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ABSTRACTS OF POSTERS

Periodic solutions of anisotropic Euler-Lagrange equations in $\mathbf{W^1_T L^G}([-T,T],\mathbb{R}^N)$

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Using the Mountain Pass Theorem, we establish the existence of periodic solution for Euler-Lagrange equation with Lagrangian $L : [-T,T] \times \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}$ given by $L(t,x,v) = G(v) + V(t,x) + \langle f, x\rangle$, where $G : \mathbb{R}^N \to \mathbb{R}$ is an anisotropic G-function and potential $V(t,x)$ satisfies some growth conditions.

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On solvability of nonlinear eigenvalue problems for degenerate PDEs of elliptic type

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Fine approximation of equivariant locally Lipschitz functions

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First, we make some observations on the symmetrization procedure applied to a set-valued map and take glance at C^1-fine approximations on Banach space. We concern on the existence of smooth approximations of equivariant locally Lipschitz functions. Their gradient should be a graph-approximation of the Clarke generalized gradient.

These are technical results which will be applied to various differential inclusions then.

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Variational formulation and computational simulations of mechanical thermal contact problem

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Modeling of contact between deformable bodies has great practical value. These problems occur in many different settings. Some examples from automotive industry are contact between brake pads and rotors or between pistons and cylinders. The classical approach to solve presented problem requires the use of the theory of variational or hemivariational inequalities. It was developed and presented in [2]. The adaptation of this theory to solve mechanical contact problems can be found for example in [3]. A good example of a publication summarizing recent progress in this field is [4].

In many cases in the field of contact mechanics existence and uniqueness of solution to particular problem was proved, but numerical estimation of finite element method error was not conducted. There are some examples of scientific papers and publications that provide this estimation for particular cases, for example [5], [6] and [7]. Still, a wide variety of problems is not covered.

This poster is based on a paper [1] and presents the formulation and the finite element approximation of a quasi-static thermoviscoelastic problem which describes frictional contact between a deformable body and a rigid foundation. The contact is modeled by normal damped response condition whereas the friction is described by the Coulomb law of dry friction.

The weak formulation of the model consists of a coupled system of the variational inequality for the displacement and the parabolic equation for the temperature. The main aim of this work is to present a fully discrete scheme for numerical approximation together with an error estimation of a solution to this problem. Finally, computational simulations are performed to illustrate the mathematical model.

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Periodic solutions for second-order systems with anisotropic growth

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We show an existence of periodic solution to the class of second-order Lagrange equation with anisotropic growth. We use the direct method of the calculus of variation i.e. we obtain solution minimizing certain functional I on anisotropic Orlicz–Sobolev space under conditions which ensure that I is weakly lsc and coercive.

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Determination of the properties of semiconductor heterostructures by simulation based on the drift-diffusion model with the Composite Discontinuous Galerkin Method

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Numerical simulations of semiconductor devices with the drift-diffusion model are discussed. Discretization of the linear elliptic ordinary/partial differential equations in $\mathbb{R}^d$, $d \in \{1,2\}$ are presented with two variants of the Composite Discontinuous Galerkin Method. Analysis of these variants in context of discretization of the nonlinear Poisson equation is presented. Error estimates of both discretizations are shown.

We also discuss these methods in context of simulations of luminescent devices based on gallium nitride and its mixed compounds with drift-diffusion model, using van Roosbroeck equations:

$$\begin{aligned}
-\nabla\left(\varepsilon \nabla u^*\right) + e^{u^*-v^*} - e^{w^*-u^*} &= k_1,\\
-\nabla(\mu_n e^{u^*-v^*}\nabla v^*) - Q(u^*,v^*,w^*)(e^{w^*-v^*}-1) &= 0,\\
-\nabla(\mu_p e^{w^*-u^*}\nabla w^*) + Q(u^*,v^*,w^*)(e^{w^*-v^*}-1) &= 0.
\end{aligned}$$

Here u, v, w are unknown functions, $\varepsilon, \mu_n, \mu_p, k_1$ correspond to material parameters and $Q(x,u,v,w)$ is an operator depending on the semiconductor material. The proposed methods account for lower regularity of solutions on the interfaces between layers of different materials.

We present examples of numerical simulations and convergence analysis based on numerical experiments, which are performed for discretization of the nonlinear Poisson equation alone (related to physical equilibrium case) as well as for coupled van Roosbroeck equations (non-equilibrium case).

It is also demonstrated that these methods may be used in determination of physical properties of the luminescent devices. Examples of such simulations are also discussed.

Analysis of rigid-viscoplastic material in contact mechanics

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The mathematical model for the unilateral contact of a rigid-viscoplastic body with a rigid foundation is studied. The so-called classical Signorini boundary condition is analyzed and the contact is assumed to be frictionless. The problem is quasistatic and its variational formulations are discussed. We consider a constitutive law of rigid-viscoplasticity in d-dimensions ($d = 1, 2, 3$) of the form

$$\varepsilon(\dot{u}) = \widetilde{\mathcal{G}}(\sigma, \varepsilon(u))$$

where

$$\widetilde{\mathcal{G}}(\sigma, \varepsilon(u)) = -\mathcal{E}^{-1}\mathcal{G}(\sigma, \varepsilon(u)),$$

$\mathcal{E}^{-1}$ denotes the inverse of the linear elasticity operator $\mathcal{E}$ and the viscoplasticity operator $\mathcal{G}$ is in general nonlinear. Moreover, $\mathcal{E}$ and $\mathcal{G}$ are related to material constitutive functions. The theorems on existence and uniqueness results for the above model have been analyzed.

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On some boundary value problem of Robin type

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In my work I consider a boundary value problem of Robin type. Especially I consider problems with p-Laplacian operator. I try to generalize known results for p-Laplacian operator on other operators.

Heteroclinic solutions of Allen–Cahn type equations with a general elliptic operator

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Author considers a generalization of Allen–Cahn type equation in divergence form $-\mathrm{div}\,(\nabla G(\nabla u(x,y))) + F_u(x,y,u(x,y)) = 0$. This is more general than the usual Laplace operator. The existence and regularity of heteroclinic solutions is proved under standard ellipticity and m-growth conditions. Methods introduced in [2] and used in [1] for the Allen–Cahn equation are modified to a more general elliptic operator.

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